## Life Expectancy and Lifespan Equality: A Dynamic Long Run Relationship

José Manuel Aburto, Ugofilippo Basellini, Søren Kjærgaard \& James W. Vaupel

$23^{\text {rd }}$ May 2017

## Introduction

- Background:
- Life expectancy at birth $\left(e_{0}\right)$ is one of the most widely used measures to summarize population health.
- Most countries have improved in this indicator. Record $e_{0}$ has steadily increased by 2.5 years every decade.
- However, it conceals variation in lifespans or lifespan equality.


## Introduction

- Background:
- Life expectancy at birth $\left(e_{0}\right)$ is one of the most widely used measures to summarize population health.
- Most countries have improved in this indicator. Record $e_{0}$ has steadily increased by 2.5 years every decade.
- However, it conceals variation in lifespans or lifespan equality.
- What is lifespan equality?


## Introduction

- Background:
- Life expectancy at birth $\left(e_{0}\right)$ is one of the most widely used measures to summarize population health.
- Most countries have improved in this indicator. Record $e_{0}$ has steadily increased by 2.5 years every decade.
- However, it conceals variation in lifespans or lifespan equality.
- What is lifespan equality?
- Dimension that expresses a fundamental difference in survivorship among individuals.


## Introduction

- Background:
- Life expectancy at birth $\left(e_{0}\right)$ is one of the most widely used measures to summarize population health.
- Most countries have improved in this indicator. Record $e_{0}$ has steadily increased by 2.5 years every decade.
- However, it conceals variation in lifespans or lifespan equality.
- What is lifespan equality?
- Dimension that expresses a fundamental difference in survivorship among individuals.
- It addresses the growing interest in health inequalities and its linkage with social behavior.

Why studying lifespan equality is important?
Danish Females


Strong association between life expectancy and lifespan equality
Life expectancy $\left(e_{0}\right)$ vs lifespan equality $(\eta)$


## Non-stationary series



## If non-stationarity $\longrightarrow$ risk of misleading results

Life expectancy ( $\mathrm{e}_{0}$ ) vs lifespan equality $(\eta)$


Stochastic properties suggest analyzing both in first differences
Changes in life expectancy and lifespan equality


## General idea of the model

Life expectancy $\left(e_{0}\right)$ vs lifespan equality ( $\eta$ )


## General idea of the model

Life expectancy $\left(e_{0}\right)$ vs lifespan equality ( $\eta$ )


## General idea of the model

Life expectancy $\left(e_{0}\right)$ vs lifespan equality ( $\eta$ )


## General idea of the model

Life expectancy $\left(e_{0}\right)$ vs lifespan equality ( $\eta$ )


General idea of the model
Life expectancy ( $e_{0}$ ) vs lifespan equality ( $\eta$ )


## General idea of the model

Life expectancy $\left(e_{0}\right)$ vs lifespan equality ( $\eta$ )


## Cointegration analysis

Two-dimensional VAR model in its equilibrium correction (VECM) form:

$$
\Delta Z_{t}=\sum_{i=1}^{k-1}\left\ulcorner\Delta Z_{t-i}+\alpha \beta^{\prime} Z_{t-1}+\mu+\psi D_{t}+\epsilon_{t}\right.
$$

where:

- $\Delta$ first difference operator
- $Z_{t}$ vector of stochastic variables, $e_{0}$ and $\eta$
- $D_{t}$ vector of deterministic variables (e.g. linear trends)

Data comes from HMD, over 8500 lifetables for 44 countries

## Lifespan equality measures

Three measures were used:

$$
\begin{equation*}
\eta=-\log \left(\frac{-\int_{0}^{\omega} \ell(x) \ln \ell(x) \mathrm{d} x}{\int_{0}^{\omega} \ell(x) \mathrm{d} x}\right)=-\log \left(\frac{e^{\dagger}}{e_{0}^{o}}\right) \tag{1}
\end{equation*}
$$

$\eta$ based on Keyfitz' entropy used in Colchero et al 2016.

$$
\begin{equation*}
\bar{\ell}=-\log \left(1-\frac{-\int_{0}^{\omega} \ell^{2}(x) \mathrm{d} x}{\int_{0}^{\omega} \ell(x) \mathrm{d} x}\right)=-\log (G) \tag{2}
\end{equation*}
$$

$\bar{\ell}$ a variant of the Gini coefficient.

$$
\begin{equation*}
c v=-\log \left(\frac{\sqrt{\int_{0}^{\omega}\left(x-e_{0}^{o}\right)^{2} f(x) \mathrm{d} x}}{\int_{0}^{\omega} \ell(x) \mathrm{d} x}\right)=-\log \left(\frac{\sigma}{e_{0}^{o}}\right) \tag{3}
\end{equation*}
$$

cv a variant of the coefficient of variation.

## Long run relationship [Johansen's trace test]



## Speed of adjustment towards long term equilibrium



Include the age dimension Reducing deaths at any age increases $e_{0}$; for $\eta$, it depends whether deaths occur before or after $a^{i}$


Threshold age $a^{\eta}$


## Decomposition method

Model of continuous change: analysis based on the assumption that covariates change continuously along an actual or hypothetical dimension. [Horiuchi et al 2008 Demography; Caswell 2010 Jourral of Ecology]

The effect of the $i$-th age group death rate on the change in $e_{0}$ and $\eta$ from period $t$ to $t+1$ can be calculated as

$$
\begin{equation*}
c_{i}=\int_{m_{i}(t)}^{m_{i}(t+1)} \frac{\partial e_{0}(t)}{\partial m_{i}(t)} d m_{i}(t) \tag{4}
\end{equation*}
$$

Then we calculated contributions below and above the threshold age to changes in life expectancy and lifespan equality.

## Age-specific contributions



Changes above the threshold age


## Summary and conclusions

- Strong association between changes in $e_{0}$ and $\eta$.
- We found evidence of a long term equilibrium.
- Even if in the short term they diverge from each other, there is a correction mechanism that bring them together again.
- To some extent mortality improvements below threshold age are driving the relationship.


## Thanks for your attention.

## Comments and/or questions?

Normalized $(\eta=1)$ long run coefficient for $e_{0}$


## Can we talk about causality?

- Granger causality $\longrightarrow$ Because $e_{0}$ and $\eta$ cointegrate at least Granger causality exists in one direction.[Caution!]
- Just a potential causality, does not take into account latent variables.
- Temporal precedence: a cause precedes its effects in time
- Instantaneous causality: test non-zero correlation between error processes of the cause and effect variables. In $90 \%$ of the cases we reject the $H_{0}=$ no instantaneous causality


## long run relationship



