## Age Heaping in Population Data of Emerging Countries

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5th HMD symposium, Harnack Haus, Berlin, May, 2019





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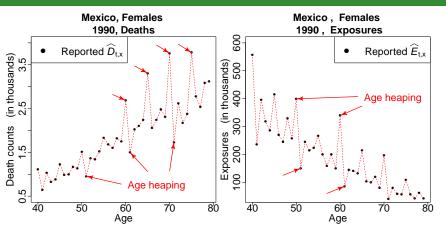
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#### Motivation.



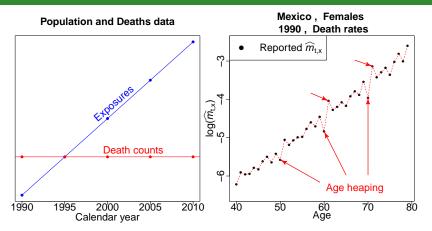
Age Heaping occurs when people misreport age.







#### Motivation.



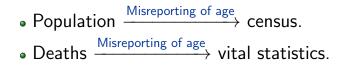


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 $\bullet \text{ Mortality analyses } \xrightarrow{\text{High quality mortality data}} \text{HMD}.$ 

However, in many countries population and deaths data can be somewhat unreliable.







#### Main Objective

• Develop mortality models for countries where population data is affected by age heaping.

Application: Reported data  $\rightarrow$  Smoothed HMD  $\rightarrow$  International Reinsurance.





## MLE and Bayesian approaches

We design a model taking into account two dimensional data. Hence, we consider the data by age x and calendar years  $t \in \mathcal{T}$  for deaths, and census years  $t \in \mathcal{T}_c$ .

• 
$$\mathcal{T} = \{1990, \cdots, 2010\},\$$

•  $\mathcal{T}_c = \{1990, 1995, 2000, 2005, 2010\}.$ 



### MLE and Bayesian approaches

For any calendar year |t|,  $n_t = |t|$  is the length of the years in our data set. Where x represents the reported age and y the true age.

$$E_{t,y} \xrightarrow{\text{Age heaping}} \widehat{E}_{t,x}$$
$$D_{t,y} \xrightarrow{\text{Age heaping}} \widehat{D}_{t,x}$$







#### Model and Notation

$$\widehat{D}(t,x)\Big|\underline{m},\underline{E},\theta\sim Poisson\Big(\sum_{y}m(t,y)E(t,y)g^{D}(t,x,y)\Big), \ \forall t\in\mathcal{T}$$

$$\widehat{E}(t,x)\Big|\underline{E}, \theta \sim \textit{Poisson}\Big(\sum_{y} E(t,y)g^{E}(t,x,y)\Big), \ \forall t \in \mathcal{T}_{c}.$$

$$m(t,y) = \exp\left[a(t) + b(t)(y - \overline{y}) + c(t)\left((y - \overline{y})^2 - \sigma_y^2\right)\right].$$



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$$\ell_{E}(\theta) = \sum_{x} \sum_{t \in \mathcal{T}_{c}} \left[ \widehat{E}(t, x) \log \left( \sum_{y} E(t, y) g^{E}(t, x, y) \right) - \sum_{y} E(t, y) g^{E}(t, x, y) \right] + \text{Const}$$

 $g^* \rightarrow$  Probability that an individual report their true age (y) given that they have reported age (x) in calendar year (t),

 $\eta^* \rightarrow {\rm Captures}$  the increasing improvements across years,

 $H^* \rightarrow$  Describes "popular" ages.





Where,

$$\theta = \{\underline{E}, \underline{\eta}^{\mathsf{D}}, \underline{\eta}^{\mathsf{E}}, \underline{H}^{\mathsf{D}}, \underline{H}^{\mathsf{E}}, \underline{a}, \underline{b}, \underline{c}, \delta_{\mathsf{A}}, \mu_{\mathsf{b}}, \mu_{\mathsf{c}}\}$$

#### **Prior distributions**

 $E(t, y) \sim U(0, \infty)$  iid,

$$\eta^{\mathcal{D}}(t) \sim ext{ random walk}, \qquad \eta^{\mathcal{E}}(t) \sim ext{ random walk},$$

$$H^{D}(x) \Big| \sim \mathbf{Exp}(\lambda) \text{ iid}, \qquad \Big| H^{E}(x) \Big| \sim \mathbf{Exp}(\lambda) \text{ iid}.$$



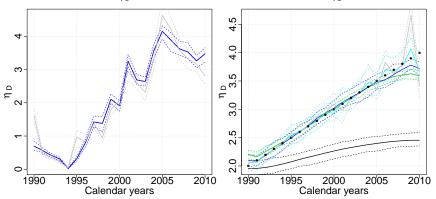


#### Actuarial Execution

## Parameter $\eta^{D}$ M-H, Mexico & Canada.

mean of  $\eta_D$  Mexico

mean of  $\eta_D$  Canada



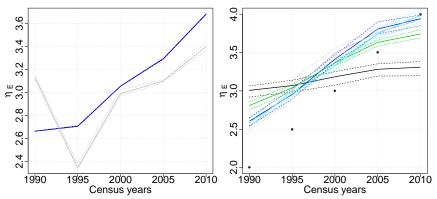




## Parameters $\eta^{E}$ M-H, Mexico & Canada.

mean of  $\eta_E$  Mexico

mean of  $\eta_E$  Canada



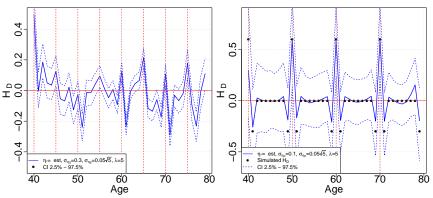




## Parameter <u>H<sup>D</sup></u> M-H, Mexico & Canada.

mean of H<sub>D</sub> Mexico

mean of H<sub>D</sub> Canada



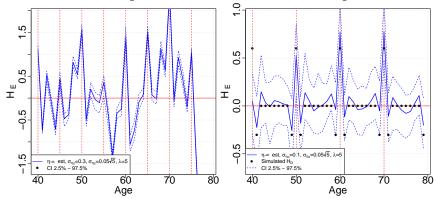




## Parameter <u>H<sup>E</sup></u> M-H, Mexico & Canada.

mean of H<sub>E</sub> Canada

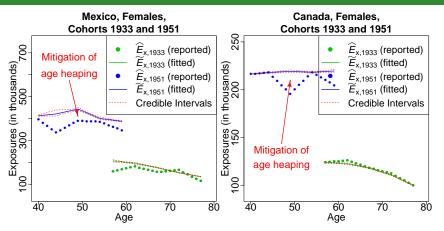
mean of H<sub>E</sub> Mexico





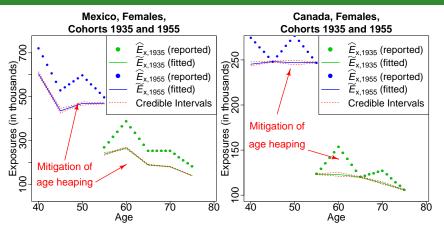


## Fitted exposures by Cohort, Mexico & Canada.





## Fitted exposures by Cohort, Mexico & Canada.





## Fitted exposures $E_{t,y}$ , Mexico & Canada 1990.

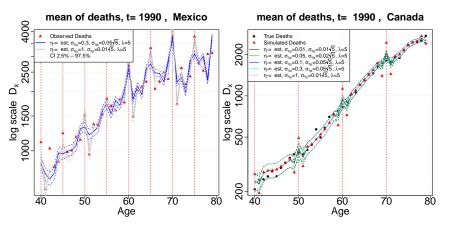
mean of exposures, t= 1990, Mexico mean of exposures, t= 1990, Canada 5e+05 200000 2e+05 ш ш log scale 1e+05 2e+ log scale 00000 True exposures Simulated heaping\_vector η-= est,  $\sigma_{ne}$ =0.01,  $\sigma_{ne}$ =0.01 $\sqrt{5}$ , λ=5  $\begin{array}{l} \eta_{-} = \text{est}, \, \sigma_{\eta_{0}} = 0.01, \, \sigma_{\eta_{0}} = 0.01/5, \, \lambda = 5\\ \eta_{-} = \text{est}, \, \sigma_{\eta_{0}} = 0.05, \, \sigma_{\eta_{0}} = 0.02/5, \, \lambda = 5\\ \eta_{-} = \text{est}, \, \sigma_{\eta_{0}} = 0.1, \, \sigma_{\eta_{0}} = 0.05/5, \, \lambda = 5\\ \eta_{-} = \text{est}, \, \sigma_{\eta_{0}} = 0.3, \, \sigma_{\eta_{0}} = 0.05/5, \, \lambda = 5\\ \eta_{-} = \text{est}, \, \sigma_{\eta_{0}} = 1, \, \sigma_{\eta_{0}} = 0.01/5, \, \lambda = 5 \end{array}$ 5e+04 Observed exposures η= est,  $\sigma_{n_0}=0.3$ ,  $\sigma_{n_0}=0.05\sqrt{5}$ ,  $\lambda=5$ η-= est,  $σ_{n_0}$ =1,  $σ_{n_e}$ =0.01 $\sqrt{5}$ , λ=5 CI 2.5% - 97.5% CI 2.5% - 97.5% 50 60 70 80 50 60 70 80 40 40 Age Age







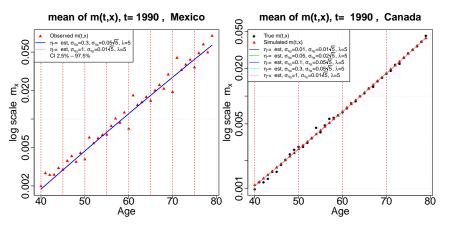
#### Fitted deaths $D_{t,y}$ , Mexico & Canada 1990.





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## Fitted exposures $E_{t,y}$ , Mexico & Canada 1990.







#### Conclusions.

- Smooth time series  $\underline{a}, \underline{b}$  and  $\underline{c} \xrightarrow{\text{Reduce age heaping}} m(t, y)$  and population. However, we do not want to smooth too much because it would destroy the natural volatility from the data.
- This model improves the quality of the Mexican data by reducing age heaping across all calendar years.
- The remaining volatility in the fitted exposures comes from the death counts.





#### Conclusions

• Parameter  $\eta^*$  reflects the improvement in the quality of the data over time. Hence, we expect  $\eta^*$  to increase over time. In other words, there is less age heaping that there use to be, say in 1990.





- Include priors for parameters regarding the true force of mortality.
- Sensitivity of choosing different priors.
- We will keep collaborating with HMD to see how their approach can be adapted to Mexican data for producing complete life table series, which is also relevant to international reinsurance.







# Thank You!

# Questions?





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